Baryonic Corrections to Superpotentials from Perturbation Theory

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Abstract

We study the corrections induced by a baryon vertex to the superpotential of SQCD with gauge group $SU(N)$ and N quark flavors. We first compute the corrections order by order using a standard field theory technique and derive the corresponding glueball superpotential by "integrating in" the glueball field. The structure of the corrections matches with the expectations from the recently introduced perturbative techniques. We then compute the first non-trivial contribution using this new technique and find exact quantitative agreement. This involves cancellations between diagrams that go beyond the planar approximation.

Important progress has recently been made in computing the glueball superpotentials for a very large class of $\mathcal{N}=1$ gauge theories. In [1], Dijkgraaf and Vafa have proposed a simple technique for computing the effective superpotential for the glueball field $S=-\frac{1}{32\pi^2}\mathrm{tr}\mathcal{W}^\alpha\mathcal{W}_\alpha$ by using a matrix model. This conjecture has more recently been proven in the work of [2] where it was shown that the matrix model technique gives the right result for a class of theories admitting a planar expansion a la 't Hooft [3]. Namely, for theories with adjoint matter, only planar (closed) diagrams contribute.

The conjecture can be extended to include matter in the fundamental representation [4]. As long as there are fields in the adjoint the topological expansion is still possible, taking also into account planar diagrams with one boundary.

However, the work of [2] went further than the matrix model analogy and gave a general expression for the glueball superpotential in terms of a field theory of chiral superfields even in the case where the usual planar expansion is invalid.

It is the scope of this note to use and test this more general formula in the physically interesting case of SQCD (i.e. without adjoint matter). The first non-trivial interaction that can be added to the bare superpotential is a baryon vertex in a $\mathcal{N}=1$ gauge theory with gauge group $SU(N_c)$ and $N_f=N_c\equiv N$ quark flavors. Although the structure of the interaction does not allow for a planar expansion we show, to first non-trivial order, that a cancellation similar to the one taking place in [2] also occurs here pointing to a larger range of applicability of the new techniques.

We consider a $SU(N)$ gauge theory with N quark flavors and a tree level superpotential

$$W_{tree}=m\mathrm{tr}Q\tilde{Q}+b\det Q+\tilde{b}\det\tilde{Q}. \tag{1}$$

The quark fields Q and $\tilde{Q}$ are considered as $N\times N$ matrices and given the same bare mass m. The coupling constants of the baryon vertices are denoted by b and $\tilde{b}$. The meson and baryon operators are thus

$$M_i^j=Q_i^a\tilde{Q}_a^j,\quad B=\det Q,\quad \tilde{B}=\det\tilde{Q}. \tag{2}$$

Before adding W_{tree}, the theory has N massless quarks and holomorphic scale $\hat{\Lambda}$. At the quantum level, there is no effective superpotential but the moduli space receives corrections [5]. After we add W_{tree} and integrate out the massive fields, the theory at low energies is pure Super Yang-Mills with holomorphic scale Λ given by $\Lambda^3=m\hat{\Lambda}^2$.

It is easy to obtain the corrections to the superpotential of the low energy SYM theory by integrating out the composite fields. First we impose the condition on the quantum moduli space [5] by writing the superpotential of the high energy theory with the help of a Lagrange multiplier ξ:

$$W_{high} = \xi(\hat{\Lambda}^{2N} - \det M + B\tilde{B}). \tag{3}$$

Then we integrate out M, B and $\tilde{B}$ (and ξ) from:

$$W_{tot} = W_{high} + W_{tree}. \tag{4}$$

to find that ξ obeys the constraint:

$$\hat{\Lambda}^{2N} - \left(\frac{m}{\xi}\right)^{\frac{N}{N-1}} + \frac{b\tilde{b}}{\xi^2} = 0. \tag{5}$$

The effective superpotential for the low energy theory is now:

$$W_{low} = N\xi\hat{\Lambda}^{2N} + (N-2)\frac{b\tilde{b}}{\xi}. \tag{6}$$

The constraint (5) can be solved algebraically for small Ns,[1] but in general we must content ourselves with a series expansion in powers of $b\tilde{b}$:

$$W_{low} = N\Lambda^3\left(1 - \frac{1}{N}t - \frac{N-1}{2N^2}t^2 - \frac{(N-1)(4N-5)}{6N^3}t^3 + \dots\right) \tag{7}$$

where we have used the dimensionless and chargeless variable

$$t = \frac{\hat{\Lambda}^{2N-4}b\tilde{b}}{m^2} = \frac{\Lambda^{3(N-2)}b\tilde{b}}{m^N}. \tag{8}$$

To compare with the results of [1] and [2] we need to integrate in [6] the glueball field to obtain, for $\beta = \frac{b\tilde{b}}{m^N}$:

$$\begin{aligned} W_{glue} &= NS(-\log(S/\Lambda^3) + 1) \qquad (9) \\ &\quad -\beta S^{N-1} - \frac{N-1}{2}\beta^2 S^{2N-3} - \frac{(N-1)(3N-4)}{6}\beta^3 S^{3N-5} + \dots \end{aligned}$$

One can already notice that the functional dependence of W_{glue} on all the variables is in agreement with the counting of [2], that is the contribution

[1] For instance, for $SU(2)$. Note that this case is trivial since the baryonic interaction is just an off-diagonal mass term, the **2** and $\bar{\mathbf{2}}$ representations being equal. Nevertheless if we insist in expanding in $b\tilde{b}$ all the following formulas hold for this case, and match the perturbative derivation, even if all the terms are proportional to S.

from diagrams with $2k$ baryon vertices will carry Nk propagators and require $Nk - 2k + 1$ factors of S to soak up the fermionic zero modes. The generic form of the numerical coefficients is also worth commenting on – note that there appear no powers of N in the denominator (contrary to (7)) pointing possibly to a simple counting argument even in the absence of a planar expansion.

We will now reproduce the first coefficient with the technique of [2] showing that we indeed find an amplitude which is topological in nature. However this comes as a result of a subtle cancellation between diagrams which would naively be thought as contributing to different orders in the large N expansion. We stress that our computation is a purely field theoretic one.

We thus compute the effective action of a theory with matter fields in a background gaugino condensate $S = -\frac{1}{32\pi^2}\mathrm{tr}\mathcal{W}^\alpha\mathcal{W}_\alpha$, following [2] (and extending to complex representations of the gauge group), as:

$$e^{-\int d^4x\, d^2\theta\, W_{eff}(S)} = \int \mathcal{D}Q\,\mathcal{D}\tilde{Q}\, e^{-\int d^4x\, d^2\theta\, [-Q(\Box - i\mathcal{W}^\alpha D_\alpha - m)\tilde{Q} + W_{int}(Q,\tilde{Q})]}, \tag{10}$$

where $W_{int}(Q, \tilde{Q})$ contains the interaction part of the tree level superpotential. It is worth noting that as far as the effective superpotential is concerned, the full path integral becomes holomorphic. This is proven in [2], and we refer to that paper for more details. From the free part of the action in (10) we can read off the propagator, which in momentum space is:

$$\Delta(p,\pi)^{ai}_{bj} = \int_0^\infty ds \left[e^{-s(p^2+m+\mathcal{W}^\alpha\pi_\alpha)}\right]^a_b \delta^i_j. \tag{11}$$

Note that we have taken for simplicity a mass matrix proportional to the identity. We can always restore generality at the end of the computation using symmetry arguments.

If now we take $W_{int} = b\det Q + \tilde{b}\det\tilde{Q}$, we have:

$$e^{-\int d^4x\, d^2\theta\, W_{eff}(S)} = \langle e^{-\int d^4x\, d^2\theta\, (b\det Q + \tilde{b}\det\tilde{Q})}\rangle \tag{12}$$

so that:

$$\begin{aligned} W_{eff}(S) \;\; &= \;\; -\sum_{n=1}^{\infty} \frac{(b\tilde{b})^n}{(n!)^2} \int d^4x_1\, d^2\theta_1\, \ldots d^4x_{2n-1}\, d^2\theta_{2n-1} \times \\ &\quad \times \langle \det Q(z_1)\ldots\det Q(z_n)\det\tilde{Q}(z_{n+1})\ldots\det\tilde{Q}(0)\rangle_c \end{aligned} \tag{13}$$

where only connected amplitudes are computed, and z represents a (chiral) superspace coordinate.

The first term in the expansion, which we are going to compute, is given by $(-b\tilde{b})$ times the $(N-1)$-loop amplitude:

$$\begin{aligned} \mathcal{A} &= \int d^4x\, d^2\theta\, \langle \det Q(z) \det \tilde{Q}(0)\rangle \qquad (14) \\ &= \int d^4x\, d^2\theta\, \epsilon_{a_1\ldots a_N}\epsilon^{b_1\ldots b_N}\langle Q_1^{a_1}\ldots Q_N^{a_N}(z)\tilde{Q}^1_{b_1}\ldots\tilde{Q}^N_{b_N}(0)\rangle \end{aligned}$$

We can now insert the propagators (11). Note that they connect only the same flavors, so that the symmetry factor of the diagram is just unity. The integrals over x and θ are going to enforce the conservation of momentum $\sum p_i = 0$ and likewise $\sum \pi_i = 0$.

The integral on the remaining bosonic momenta then gives:

$$\int \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_{N-1}}{(2\pi)^4}e^{-s_1p_1^2-\ldots-s_{N-1}p_{N-1}^2-s_N(p_1+\ldots+p_{N-1})^2} = \frac{(4\pi)^{-2(N-1)}}{(\det M(s))^2}, \tag{15}$$

where:

$$\det M(s) = s_1\ldots s_{N-1} + s_1\ldots s_{N-2}s_N + \ldots + s_2\ldots s_N. \tag{16}$$

We are also left with $N-1$ integrals on the grassmannian momenta π_i. The simple treatment for matter in the adjoint given in [2] is not applicable to our problem because of its intrinsic non-planarity. We have thus to perform the integrals explicitly:

$$\int d^2\pi_1\ldots d^2\pi_{N-1}\epsilon_{a_1\ldots a_N}\epsilon^{b_1\ldots b_N}\left[e^{-s_1\mathcal{W}^\alpha\pi_{1\alpha}}\right]^{a_1}_{b_1}\cdots\left[e^{s_N\mathcal{W}^\alpha(\pi_1+\ldots+\pi_{N-1})_\alpha}\right]^{a_N}_{b_N}. \tag{17}$$

We start by expanding the grassmannian piece of the propagator (11):

$$\left[e^{-s\mathcal{W}^\alpha\pi_\alpha}\right]^a_b = \delta^a_b - s(\mathcal{W}^\alpha)^a_b\pi_\alpha - s^2(\mathcal{W}^2)^a_b\pi^2. \tag{18}$$

We are integrating over $2N-2$ grassmannian variables a polynomial which is of degree $2N$.

Now, if we write:

$$\epsilon_{a_1\ldots a_N}\epsilon^{b_1\ldots b_N} = \delta^{b_1}_{a_1}\ldots\delta^{b_N}_{a_N} \pm (\text{permutations of } b_i), \tag{19}$$

we can easily see that the number of traces over the color indices in a particular term will be given by N minus the number of transpositions of the related permutation.

Since we take the gauge group to be $SU(N)$, we have trivially that $\mathrm{tr}\mathcal{W}^\alpha = 0$. Furthermore, as far as the first correction of the effective superpotential is concerned, we can assume that only $\mathrm{tr}\mathcal{W}^2 \propto S$ is non-zero, while all higher traces are taken to vanish, as in [2]. Note that this will not be true for higher order terms in $b\tilde{b}$: for instance in $SU(3)$ the term $\mathrm{tr}\mathcal{W}^2\mathcal{W}^2\mathcal{W}^2$ cannot be set to zero independently from $\mathrm{tr}\mathcal{W}^2$ and this will affect the calculation of the second order correction.

We thus see that the only terms contributing to the amplitude are the following: the trivial permutation, which leads to N traces and thus is proportional to $N(\mathrm{tr}\mathcal{W}^2)^{N-1}$, and the single transpositions, which have $N-1$ traces and are thus proportional to $(\mathrm{tr}\mathcal{W}^2)^{N-1}$. It is worth noting that these two contributions would arise at different orders in the large N expansion, if we were to count only the factors of N arising from the traces over the colors and not the symmetry factors due to the vertex which is also of order N.

After performing the integrals over the grassmannian momenta, the term proportional to the trivial permutation gives:

$$N(-\mathrm{tr}\mathcal{W}^2)^{N-1}(s_1^2 \ldots s_{N-1}^2 + \ldots + s_2^2 \ldots s_N^2). \tag{20}$$

The terms with one transposition, after integration, contribute like:

$$-(-\mathrm{tr}\mathcal{W}^2)^{N-1}\left[(s_1-s_2)^2 s_3^2 \ldots s_N^2 + \ldots + s_1^2 \ldots s_{N-2}^2 (s_{N-1}-s_N)^2\right], \tag{21}$$

that is, there are $\frac{1}{2}N(N-1)$ terms of the above form corresponding to every couple in the N Schwinger parameters.

Remarkably, summing these two different contributions gives the square of the determinant (16), which cancels exactly the denominator obtained from integrating over the bosonic momenta. Recalling that $S = -\frac{1}{16\pi^2}\mathrm{tr}\mathcal{W}^2$, we can write the amplitude (14) as:

$$\mathcal{A} = S^{N-1}\int ds_1 \ldots ds_N \, e^{-m\sum s_i} = \frac{1}{m^N}S^{N-1}. \tag{22}$$

Restoring the notation of Eq. (9), we have that:

$$W_{eff}(S) = NS(-\log(S/\Lambda^3)+1) - \beta S^{N-1} + O(\beta^2), \tag{23}$$

thus obtaining the first correction to the pure SYM Veneziano-Yankielowicz glueball superpotential [7].

The simplicity of the result (22) should be confronted with its non-triviality. The cancellation comes about between diagrams of two different orders, the leading one giving a potential contribution of order NS^{N-1}.

However the fact that the final integral in (22) reduces to a topological one as in the theories exemplified in [2] could indicate that some geometric picture is possible also in this case. It would be interesting to push further and see if similar cancellations arise also in higher order terms. This is currently under investigation.

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